Recent progress in understanding deconfinement and chiral restoration phase transitions

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RBRC workshop
BNL, Feb.2017

Outline

- **few comments on application of (particle-) monopoles**

- **Instanton-dyons and their ensembles in QCD-like theories**

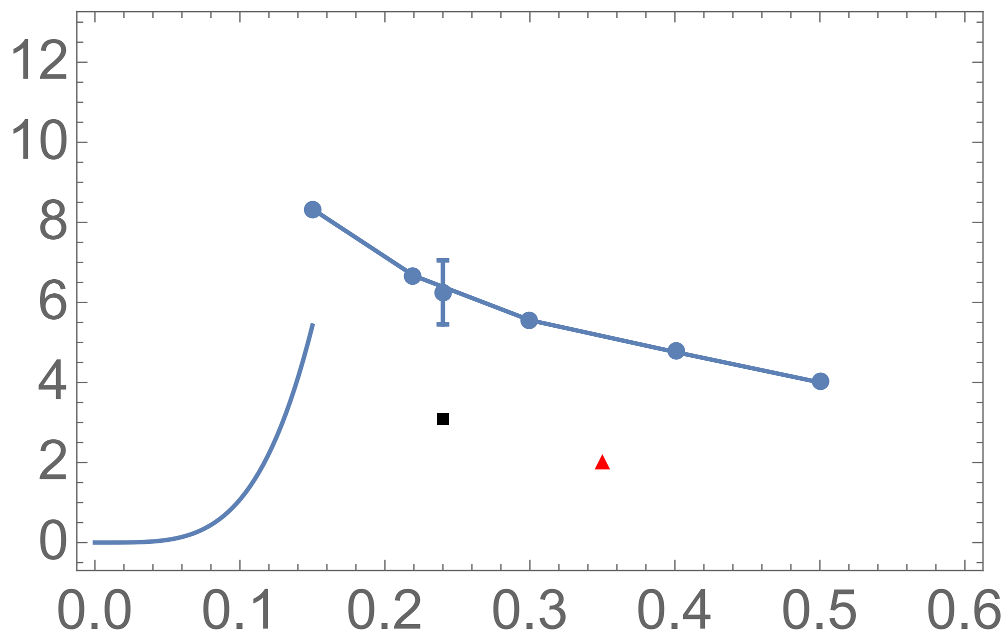
- *analytic (mean field) approach for dense ensemble (T<Tc) (1503.03058, 1503.09148 with Lui and Zahed)*

- **numerical studies at all densities: deconfinement (1504.03341 with Larsen) and chiral restoration (Nc=Nf=2)**

- **both transitions so strongly depend on quark periodicity phases that it nearly uniquely fixes the mechanism (1605.07474 with Larsen)**

applications of
(color-magnetic) monopoles-particles

monopoles and unusual features of QGP

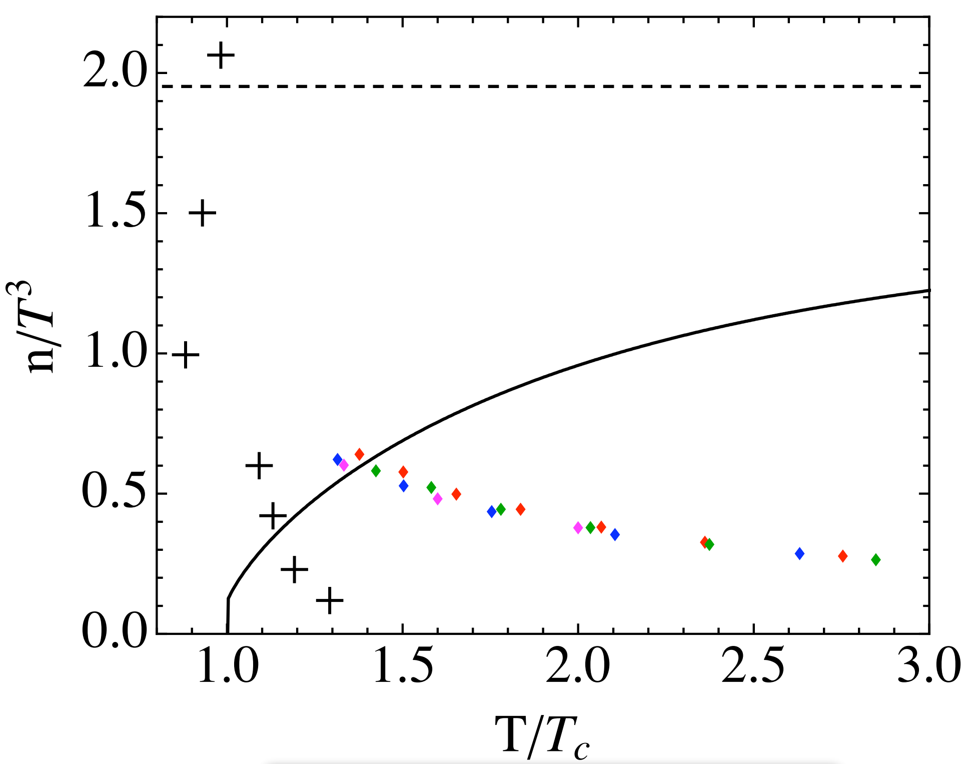


$$\frac{s}{\eta} \sim \frac{n\sigma_{transport}}{T\bar{v}}$$

Why is there a maximum at Tc?

the density of q,g dips at Tc
that of monopoles have a maximum

The entropy density to viscosity ratio s/η versus the temperature T (GeV).
The upper range of the plot, s/η = 4π corresponds to the value in infinitely strongly coupled N =4 plasma (Policastro et al., 2001).
Curve without points on the left side corresponds to pion rescatering ac- cording to chiral perturbation theory (Prakash et al., 1993).
Single (red) triangle corresponds to molecular dynamics study of classical strongly coupled colored plasma (Gelman et al., 2006a), single (black) square corresponds to numerical eval- uation (Nakamura and Sakai, 2005) on the lattice.
The single point with error bar correspond to the phenomenological value extracted from the data, see text.
The series of points connected by a line correspond to gluon-monopole scattering (Ratti and Shuryak, 2009).



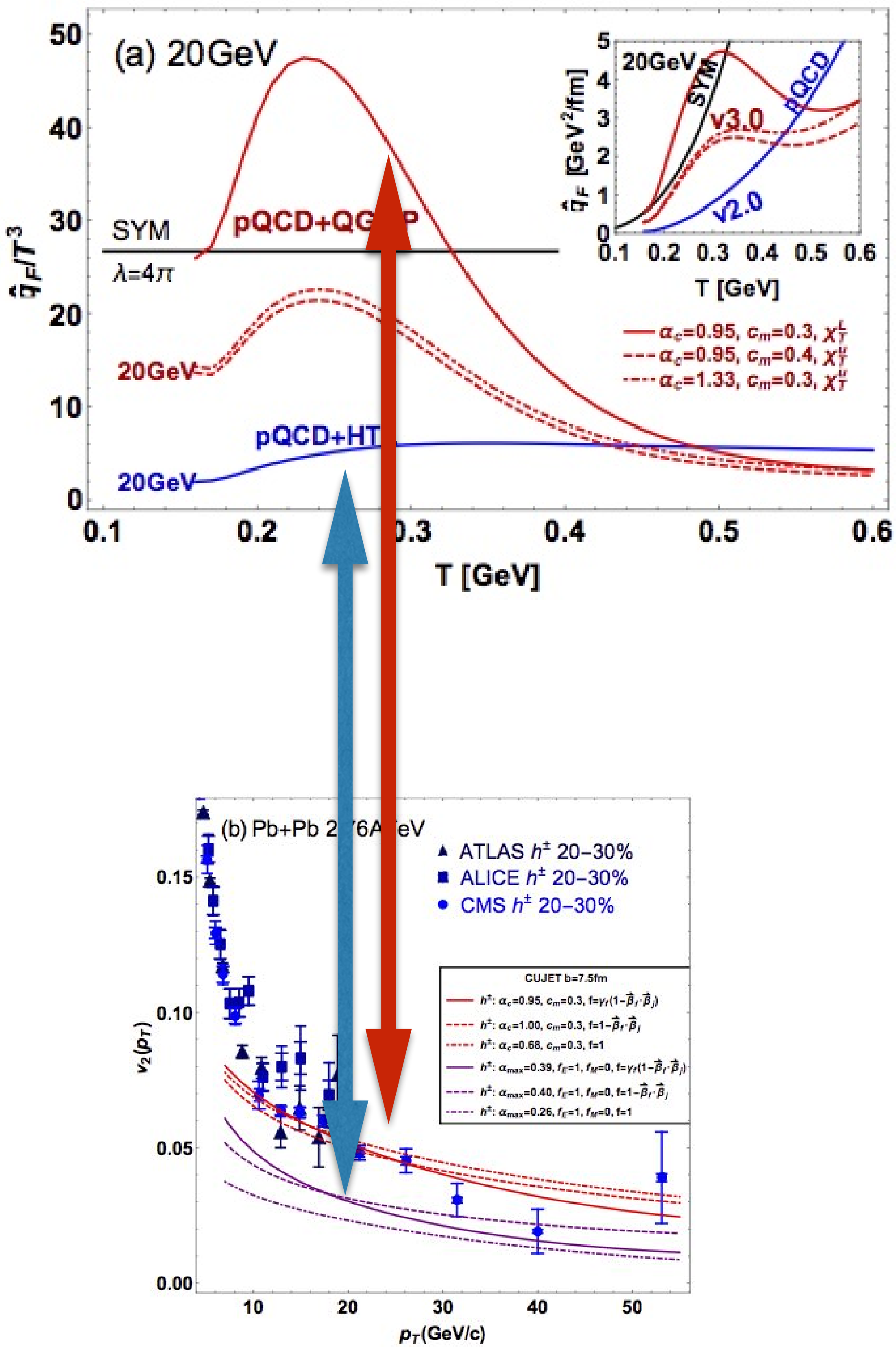
points -monopoles
line - gluons

Xu, J., J. Liao, and M. Gyulassy,

arXiv:1508.00552

another transport parameter defining jet quenching, q-hat: phenomenology require it to have a maximum near Tc
not a dip :

scattering on monopoles?



lattice monopoles do behave as particles

Thermal Monopole Condensation and Confinement in finite temperature Yang-Mills Theories

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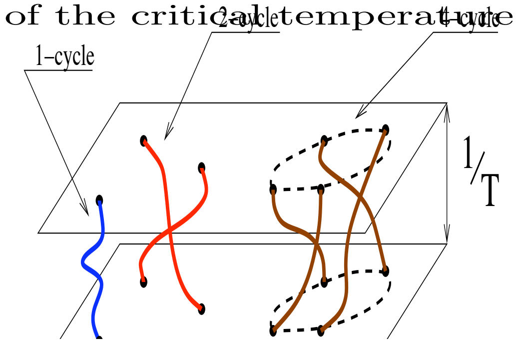
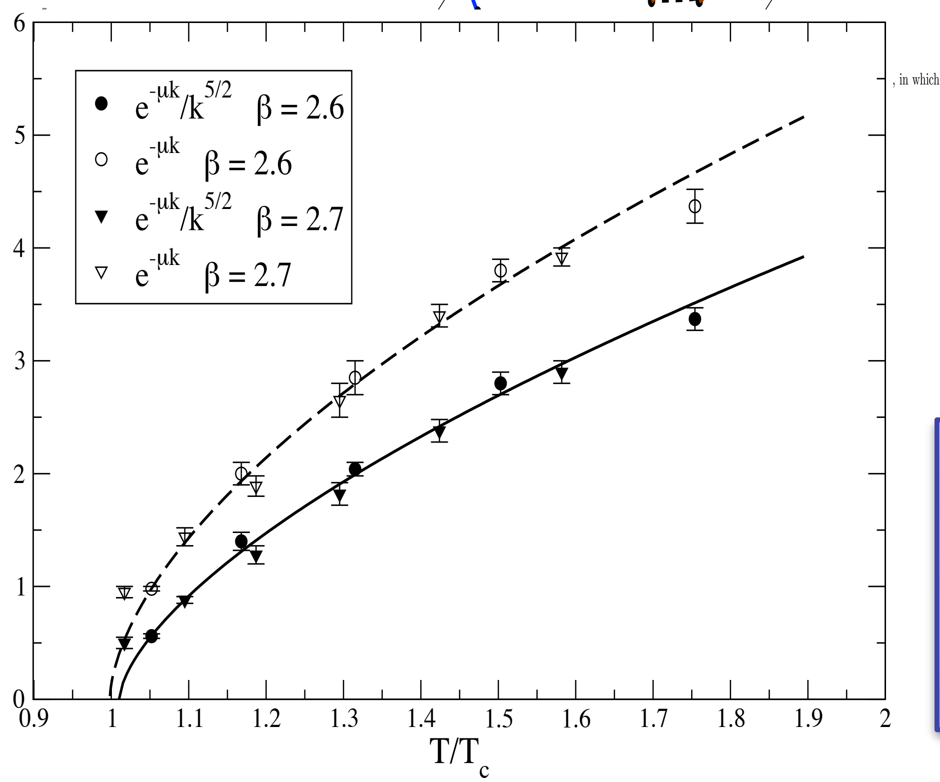
(Dated: February 22, 2010)

We investigate the connection between Color Confinement and thermal Abelian monopoles populating the deconfined phase of SU(2) Yang-Mills theory, by studying how the statistical properties of the monopole ensemble change as the confinement/deconfinement temperature is approached from above. In particular we study the distribution of monopole currents with multiple wrappings in the Euclidean time direction, corresponding to two or more particle permutations, and show that multiple wrappings increase as the deconfinement temperature is approached from above, in a way compatible with a condensation of such objects happening right at the deconfining transition. We also address the question of the thermal monopole mass, showing that different definitions give consistent results only around the transition, where the monopole mass goes down and becomes of the order of the critical temperature itself.





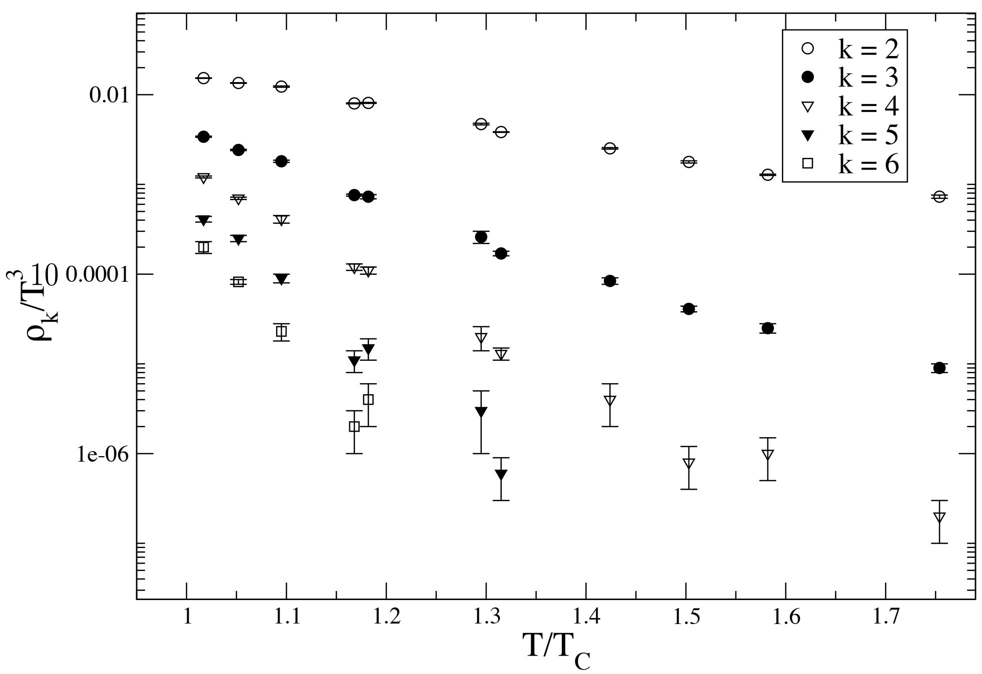


FIG. 2: Normalized densities ρ_k/T^3 as a function of T/T_c.

The lesson: monopoles at T_c, behave as He^4 =>Bose-Einstein condensation

Problems with particle-monopoles:

* not clear what is their ``Higgs scalar”
* lattice definition of monopoles depends
* on gauge
* not a semiclassical object, so no semiclassical theory

But, we believe the same physics is described by instanton-monopoles or instanton-dyons

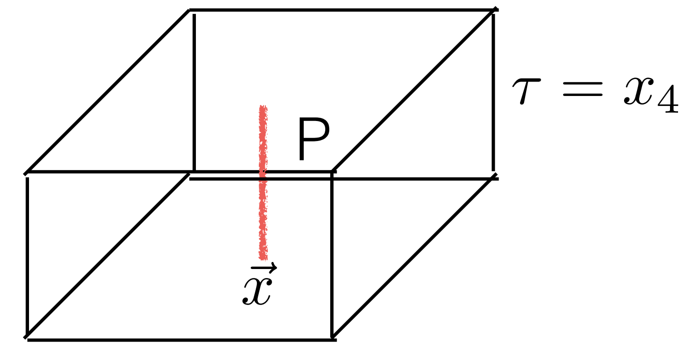
* Higgs=A_0
* there is well known semiclassical solution
* manybody semiclassical theory is being developed
* it explains both deconfinement and
* chiral transitions,
* even in QCD extended by nontrivial quark phases !

a reminder about Polyakov line on the lattice

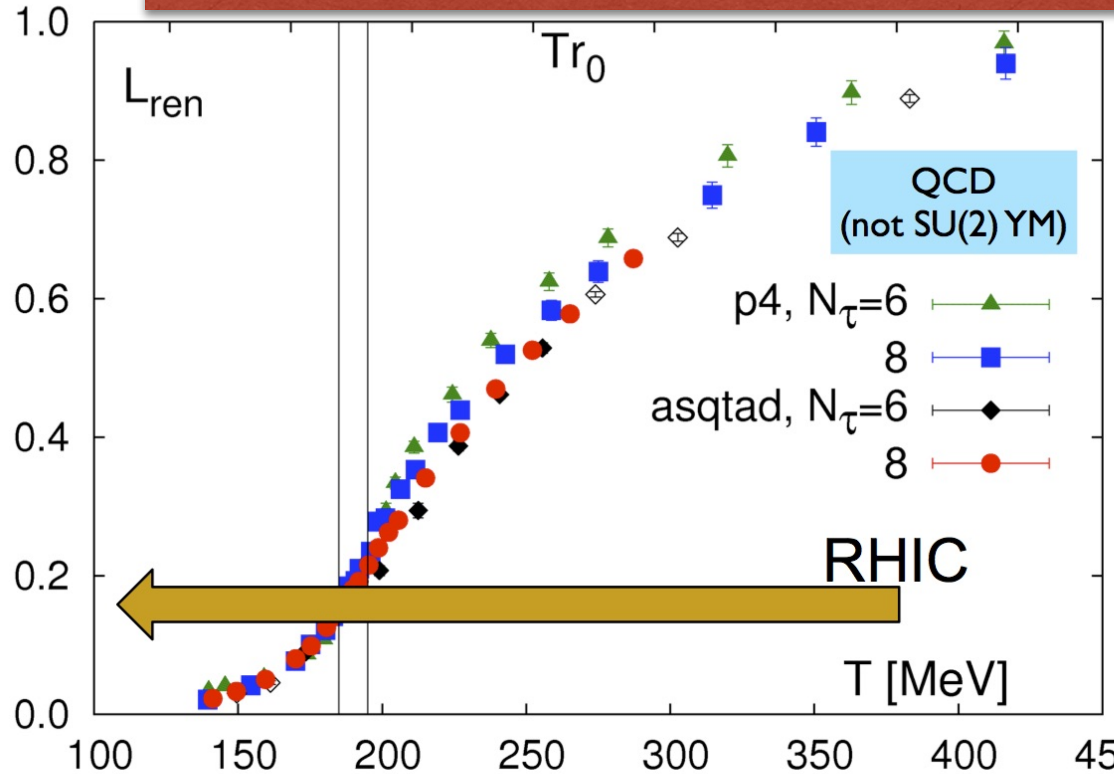
Euclidean time is defined on a Matsubara circle

and as a result there is a gauge invariant Polyakov line (holonomy)

$$L = \frac{1}{N_c} Tr P exp(i \int d\tau A_o)$$



confinement/deconfinement transition is seen as VEV of L approaching zero at a certain Tc switching off contribution of quarks and gluons to Z



the "semi-QGP" issue (Pisarski et al)

Fig. 1.4 Lattice data on the average value of the renormalized Polyakov line, as a function of the temperature T in QCD. Different points correspond to different lattice actions. Two vertical lines indicate location of the critical point, following from studies of the thermodynamical observables.

Karsch et al

the color circle
for any Nc

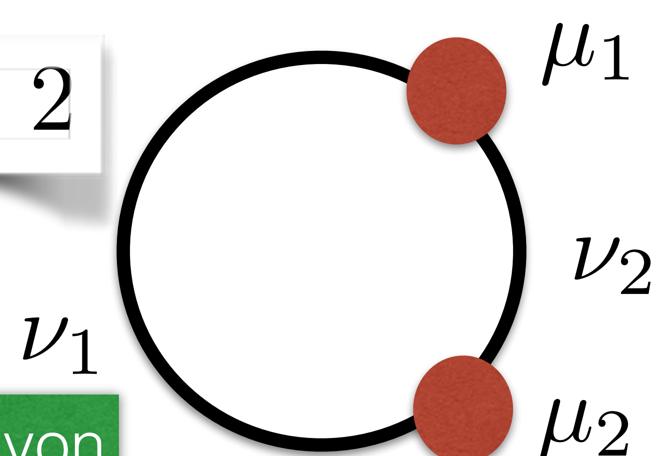
holonomy pameters

$$A_4(\infty) = 2\pi T \mathrm{diag}(\mu_1, \mu_2, \ldots, \mu_N),$$

$$\mu_1 \le \mu_2 \le \ldots \mu_N \le \mu_1 + 1, \quad \sum_{m=1}^{N} \mu_m = 0.$$

$$\nu_m \equiv \mu_{m+1} - \mu_m, \quad \sum_{m=1}^{N} \nu_m = 1.$$

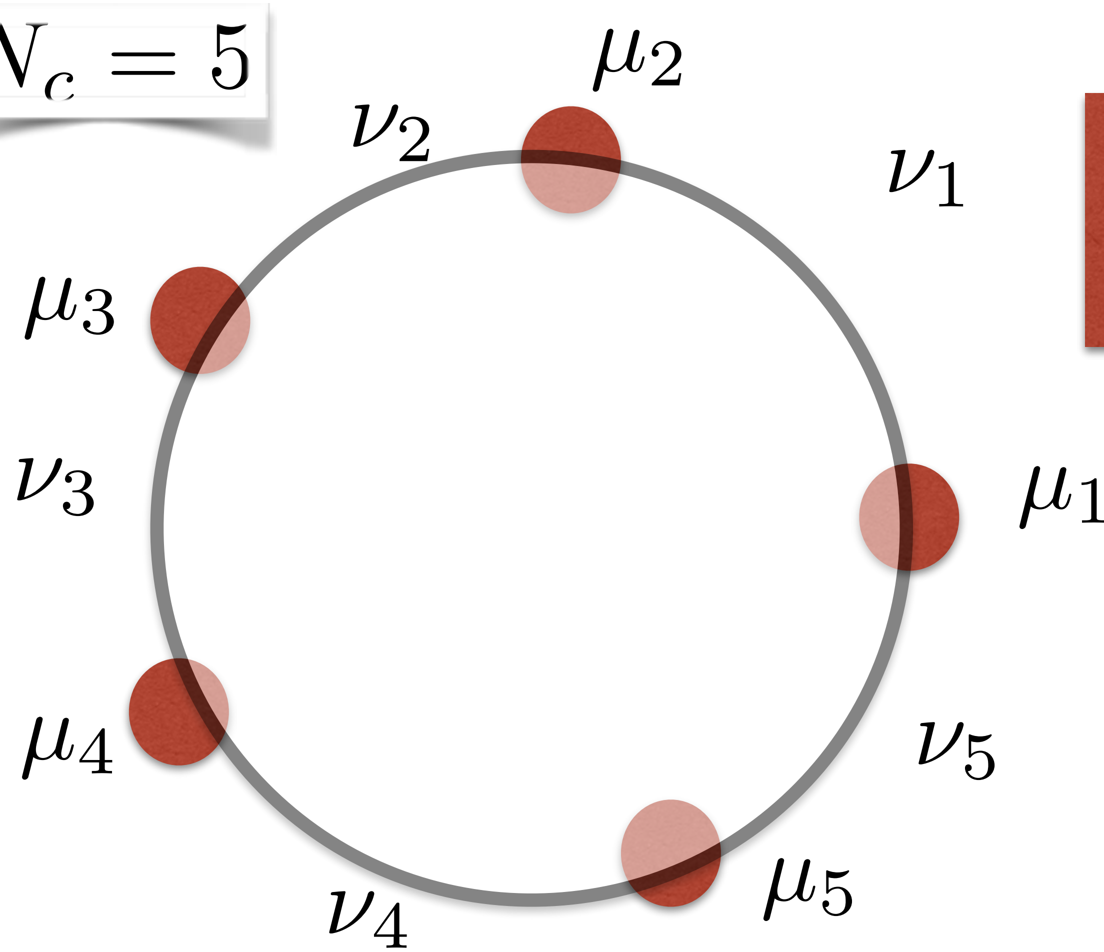
$N_c = 2$



L-dyon

M dyon
at trivial field mu_i->0 gets massless

$N_c = 5$



all nu's fill the circle
sum of dyon masses
makes full instanton

instanton-dyons ensembles
and deconfinement-chiral restoration transitions

1998

Instantons => Nc selfdual dyons (KvBLL, Pierre van Baal legacy)

<P> nonzero Polyakov line

=> <A_4>=v is non-zero

=> new solutions

Instanton liquid
4d+short range

→

Dyonic plasma
3+1d long range

instanton-dyons in SU(2)

name	E	M	mass
M	+	+	v
$\bar{M}$	+	-	v
L	-	-	$2\pi T - v$
$\bar{L}$	-	+	$2\pi T - v$

calorons=M+L are E and M neutral

TABLE I: The charges and the mass (in units of $8\pi^2/e^2T$) for 4 SU(2) dyons.

instanton-dyon = (t'Hooft-Polyakov BPS monopole)(phi=>A4)

$$
\begin{aligned}
A_4^a &= \mp n_a \, v \Phi(vr), \\
\Phi(z) &= \coth z - \frac{1}{z} \quad \xrightarrow{z\to\infty} \quad 1 - \frac{1}{z} + O(e^{-z}),
\end{aligned}
$$

$$
\begin{aligned}
A_i^a &= \epsilon_{aij} n_j \, \frac{1 - R(vr)}{r}, \\
R(z) &= \frac{z}{\sinh z} \quad \xrightarrow{z\to\infty} \quad O(z e^{-z}).
\end{aligned}
$$

M and Mbar in radial gauge

$$\vec{E} = \pm \vec{B}$$

(anti)self-dual

if A4 is "combed" up then Dirac string appears

to get L-type dyon one needs to:
(i) v->vbar=2pi T-v
(ii) make a "twist" with time-dependent matrix

$$U = \exp\left(-i\pi T x^4 \tau^3\right).$$

instantons have only top.charge
and do not interact with holonomy
but instanton-dyons do:
can their back reaction on holonomy
generate confinement?

Mitya Diakonov

possible
in a controlled setting with exp.small density
Poppitz, Schafer and Unsal
JHEP 1210 (2012) 115 arXiv:1205.0290

in it SUSY kills the perturbative Gross-Pisarski-Yaffe
holonomy potential:

can it work without SUSY?

Sulejmanpasic and ES
answered positively, but
only if there exist strong enough
dyon-antidyon repulsion

Phys.Lett. B726 (2013) 257-261
[arXiv:1305.0796]

Interacting Ensemble of the Instanton-dyons and Deconfinement Phase Transition in the SU(2) Gauge Theory

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Instanton-dyons, also known as instanton-monopoles or instanton-quarks, are topological constituents of the instantons at nonzero temperature and holonomy. We perform numerical simulations of the ensemble of interacting dyons for the SU(2) pure gauge theory. Unlike previous studies, we focus on back reaction on the holonomy and the issue of confinement. We calculate the free energy as a function of the holonomy and the dyon densities, using standard Metropolis Monte Carlo and integration over parameter methods. We observe that as the temperature decreases and the dyon density grows, its minimum indeed moves from small holonomy to the value corresponding to confinement. We then report various parameters of the self-consistent ensembles as a function of temperature, and investigate the role of inter-particle correlations.

Like in [12], instead of toroidal box with periodic boundary conditions in all coordinates, our simulations have been done on a S^3 sphere (in four dimensions), to simplify treatment of the long range Coulombic forces.

The partition function we simulate depends on several parameters, changed from one simulation set to another. Those include (i) the number of the dyons N_M, N_L; (ii) the radius of the S^3 sphere r; (iii) the action parameter S; (iv) the value of the holonomy ν, (v) the value of the Debye mass M_D; (vi) the auxiliary factor λ, which is then integrated over as explained in section IV.

$$f = \frac{4\pi^2}{3}\nu^2\bar{\nu}^2 - 2n_M \ln\left[\frac{d_\nu e}{n_M}\right] - 2n_L \ln\left[\frac{d_{\bar{\nu}} e}{n_L}\right] + \Delta f$$

Gross-Pisarski-Yaffe perturbative term +free dyons+ interaction

$\nu = 0$ *is the trivial case*

$\nu = 1/2$ *confining*

Larsen's talk at parallel session tuesday afternoon show only the “selfconsistent” input set.

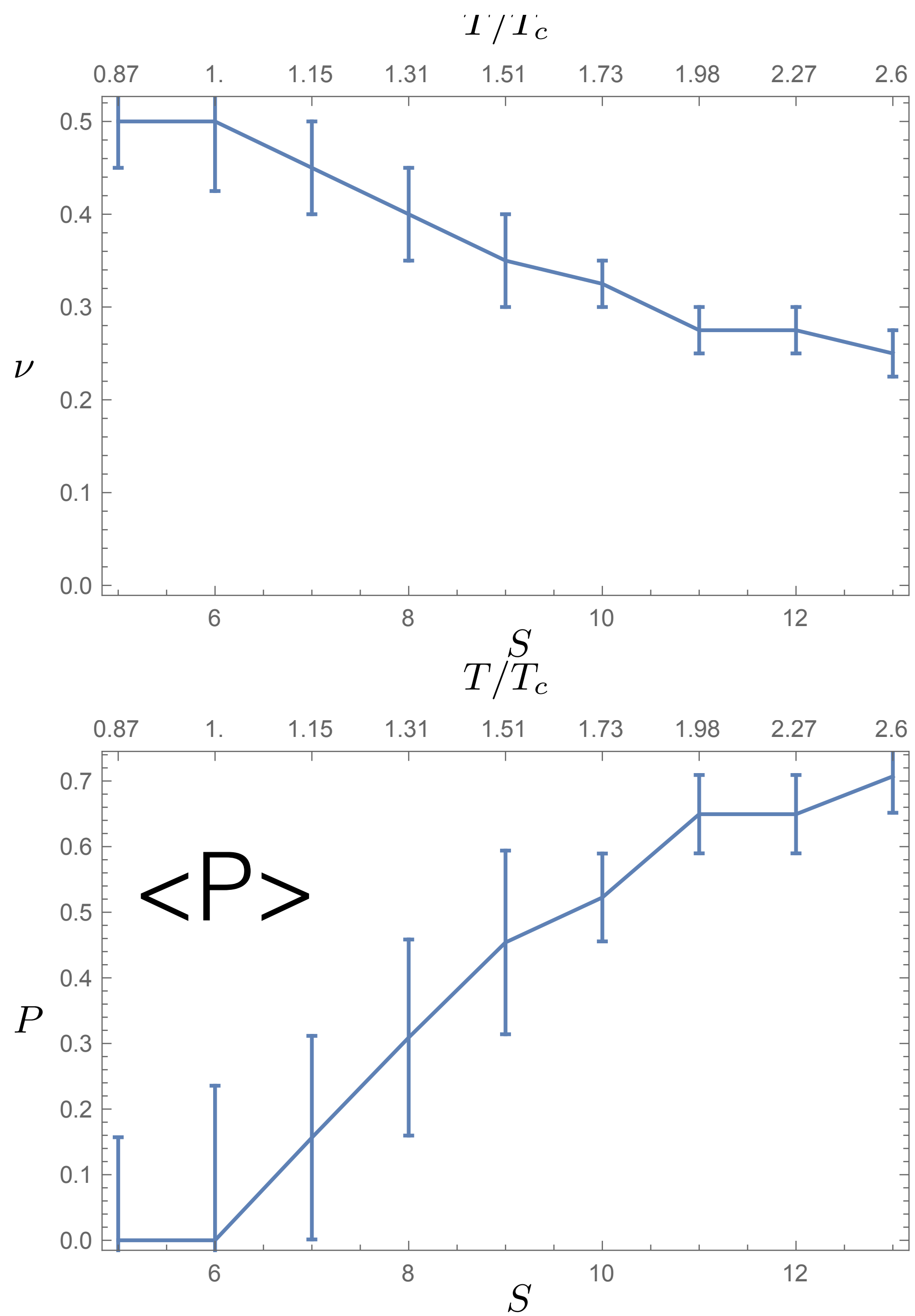

FIG. 6: Self-consistent value of the holonomy ν (upper plot) and Polyakov line (lower plot) as a function of action S (lower scales), which is related to T/T_c (upper scales). The error bars are estimates based on the fluctuations of the numerical data.

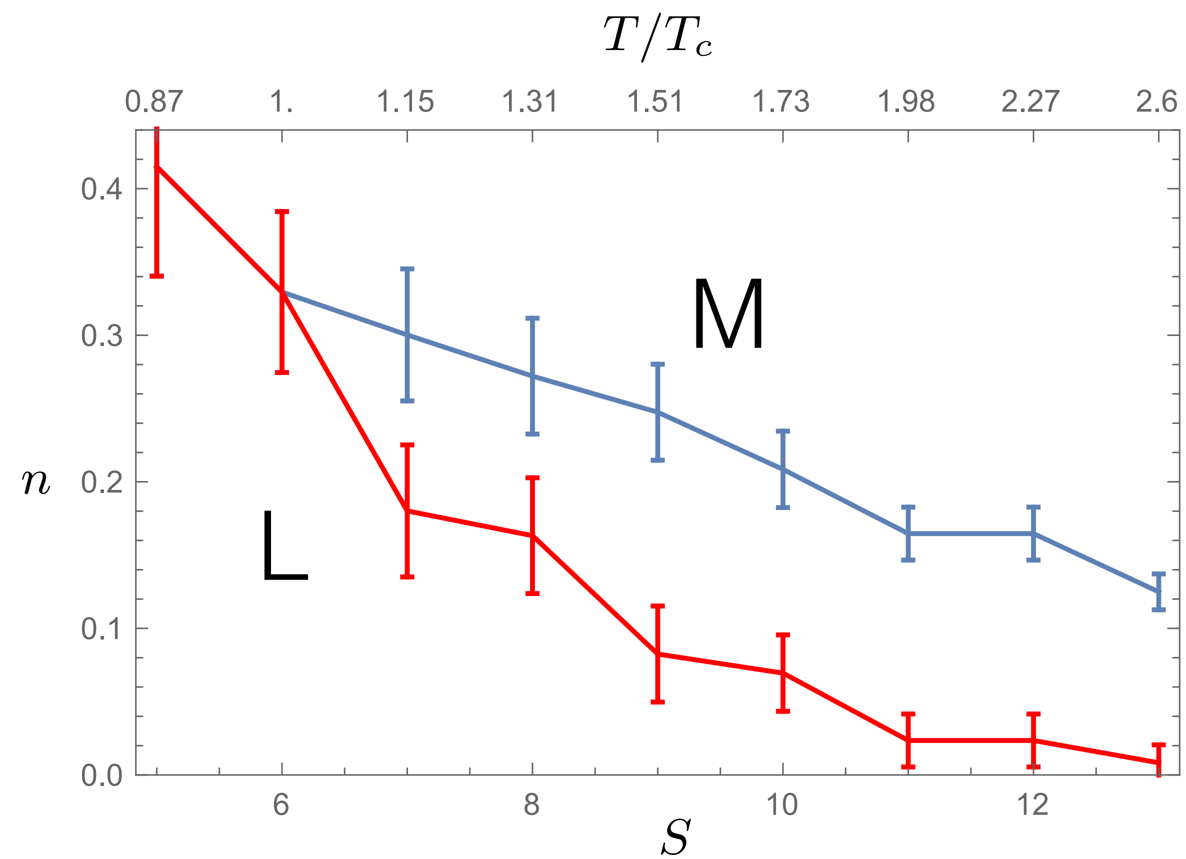

FIG. 8: (Color online). Density n (of an individual kind of dyons) as a function of action S (lower scale) which is related to T/T_c (upper scale) for M dyons(higher line) and L dyons (lower line). The error bars are estimates based on the density of points and the fluctuations of the numerical data.

confining phase is symmetric
$n_L = n_M$

$$S = \left(\frac{11N_c}{3} - \frac{2N_f}{3}\right) log\left(\frac{T}{\Lambda_T}\right).$$

Instanton-dyon Ensemble with two Dynamical Quarks: the Chiral Symmetry Breaking

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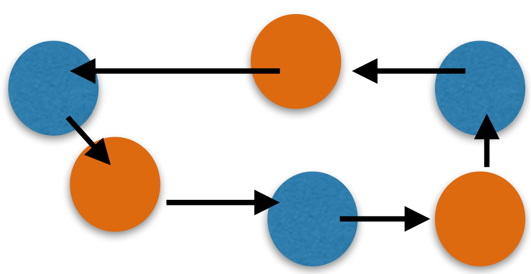
This is the second paper of the series aimed at understanding of the ensemble of the instanton-dyons, now with two flavors of light dynamical quarks. The partition function is appended by the fermionic factor, $(detT)^{N_f}$ and Dirac eigenvalue spectra at small values are derived from the numerical simulation of 64 dyons. Those spectra show clear chiral symmetry breaking pattern at high dyon density. Within current accuracy, the confinement and chiral transitions occur at very similar densities.

$$|<\bar{\psi}\psi>| = \pi\rho(\lambda)_{\lambda\to 0, m\to 0, V\to\infty}$$



high density broken chiral sym

collectivized zero mode zone

dip near zero is a finite size effect

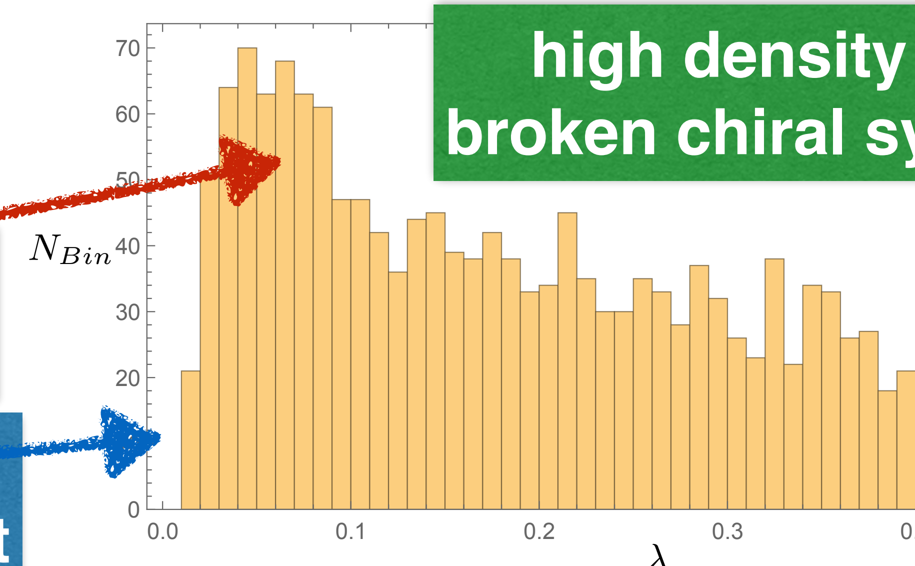


FIG. 1: Eigenvalue distribution for $n_M = n_L = 0.47$, $N_F = 2$ massless fermions.

low density chiral sym unbroken

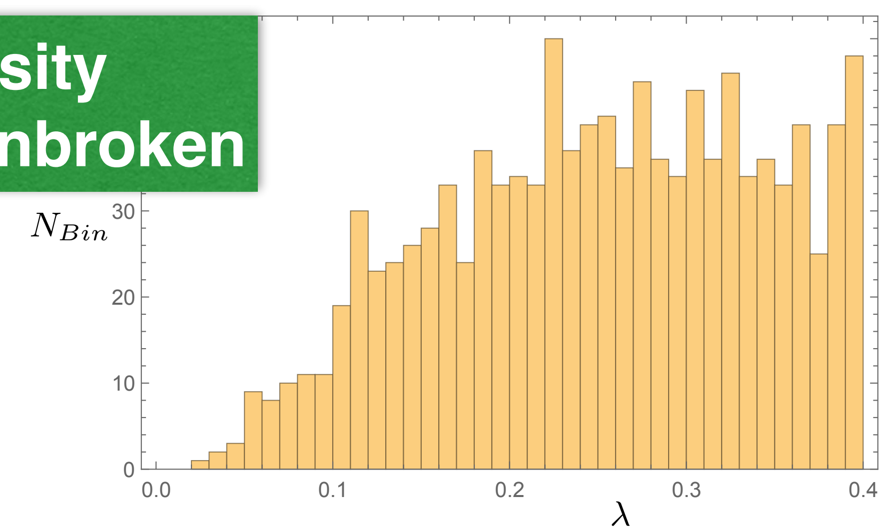
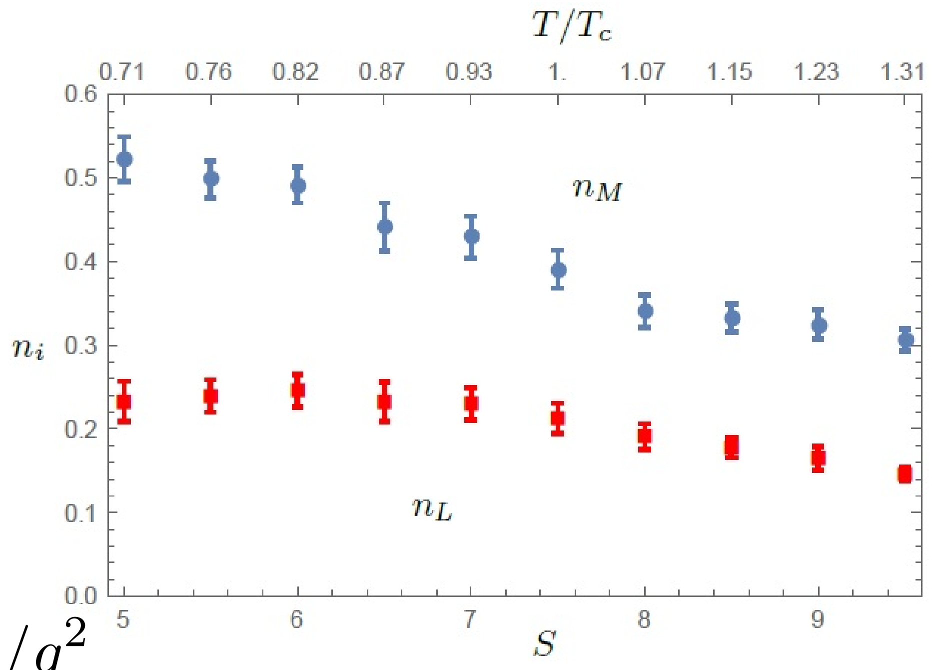


FIG. 2: Eigenvalue distribution for $n_M = n_L = 0.08$, $N_F = 2$ massless fermions.

Larsen's
talk at parallel session
tuesday afternoon

We find that the required condition for both the chiral symmetry breaking and confinement is basically sufficiently high density of the dyons.



$S = 8\pi^2/g^2$

FIG. 9: (Color online) Parameterization A: The density of the M (blue circles) and L (red squares) dyons as a function of action $S = 8\pi^2/g^2$ or temperature T/T_c.

Furthermore,
unlike in the case of pure gauge theory without quarks,
the holonomy dependence on the density is smoother.
We don't observe holonomy vanishing, and also the densities of the M and L type dyons does not become equal, even at the lowest T we studied.

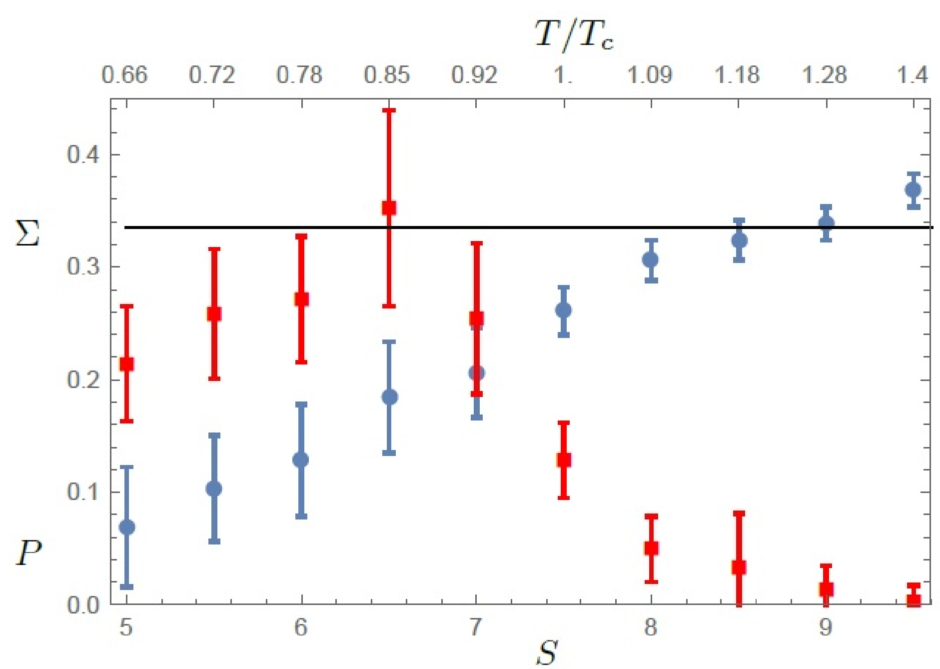
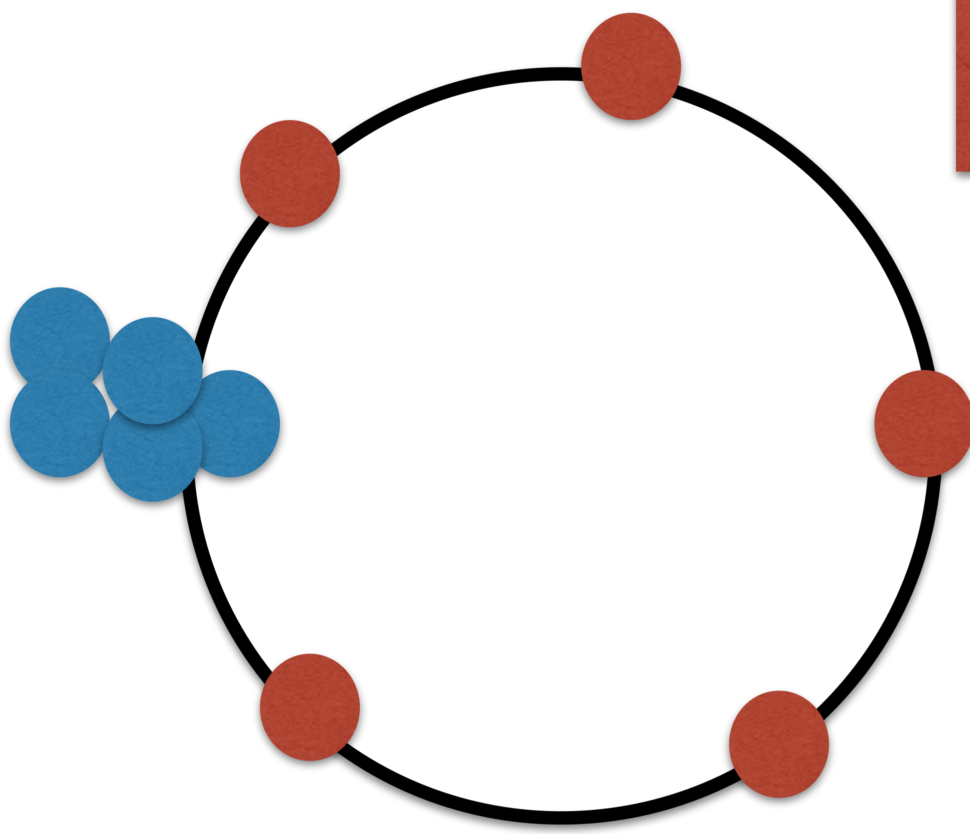


FIG. 10: (Color online) Parameterization A: The Polyakov loop P (blue circles) and the chiral condensate Σ (red squares) as a function of action $S = 8\pi^2/g^2$ or temperature T/T_c. A clear rise is seen around $S = 7.5$ for the chiral condensate. Σ is scaled by 0.2. The black constant line corresponds to the upper limit of Σ under the assumption that the entire eigenvalue distribution belong to the almost-zero-mode zone, i.e. the maximum of Σ_2.

Ordinary Nc=Nf=5 QCD



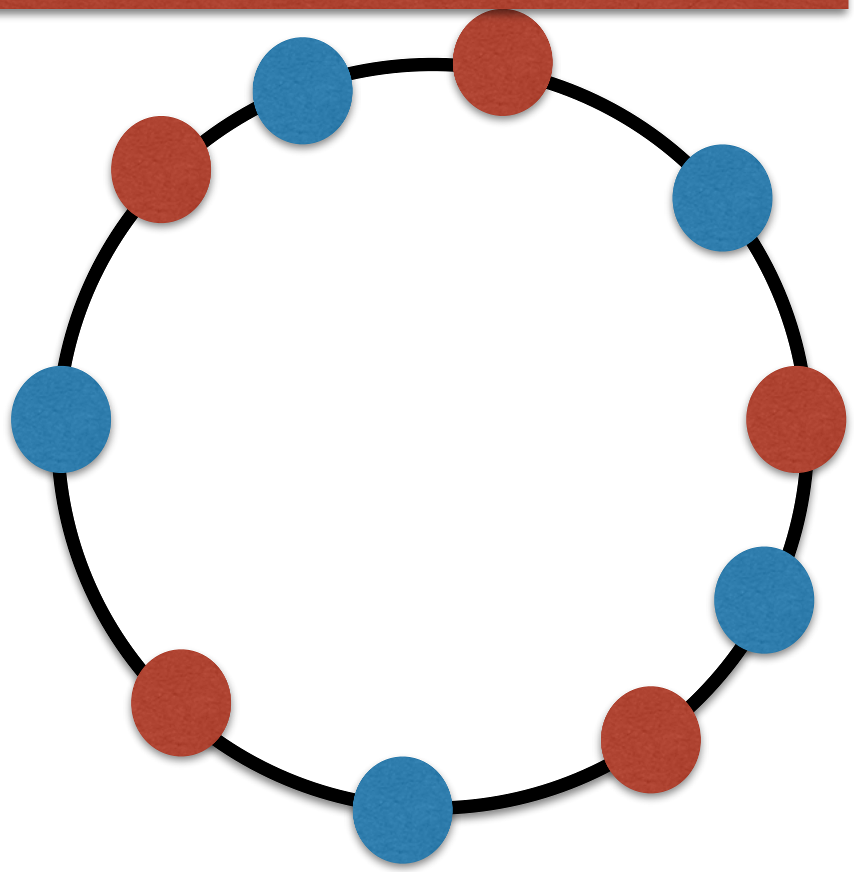
P without a trace
is a diagonal unitary matrix
=> Nc phases (red dots)

quark periodicity
phases => Nf blue dots
are in this case all =pi
quarks are fermions

as a consequence,
out of 5 types of instanton-dyons
only one has zero modes

still Nc=Nf=5 but with
“most democratic” arrangement
ZN-symmetric QCD

H. Kouno, Y. Sakai, T. Makiyama, K. Tokunaga, T. Sasaki and M. Yahiro, J. Phys. G 39, 085010 (2012).



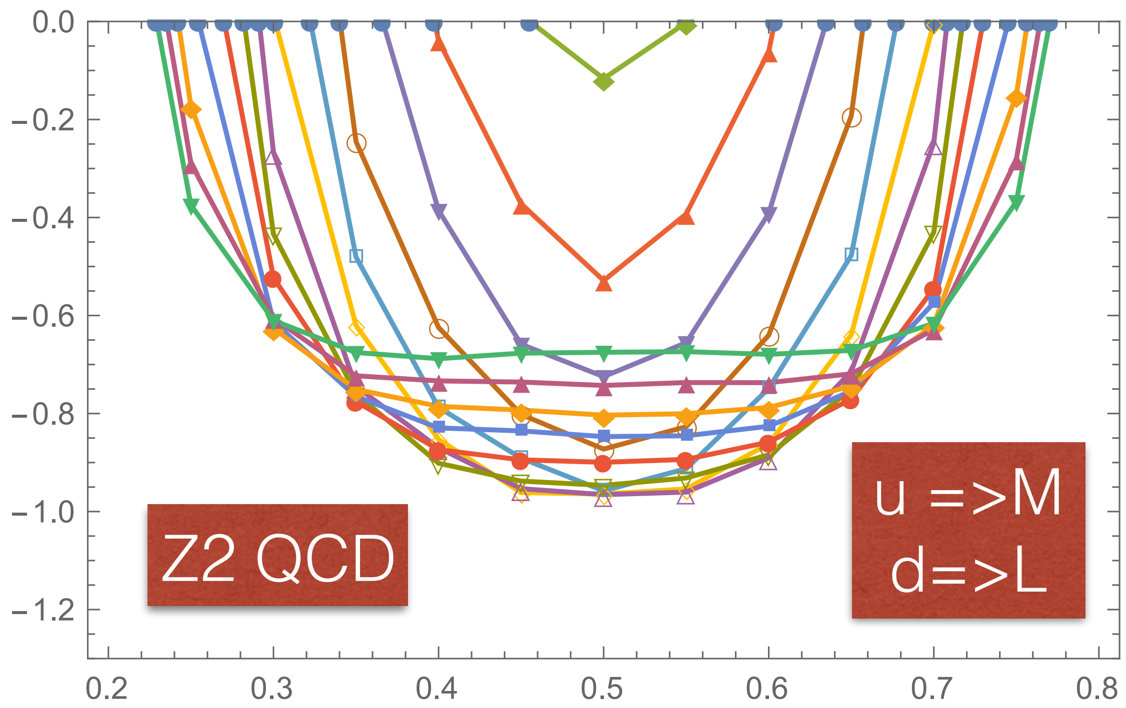
the idea: quarks can be not fermions but “anyons”

quark periodicity phases => Nf blue dots are in this case flavor-dependent (but no connection to instanton-dyons in this work, but PNJL)

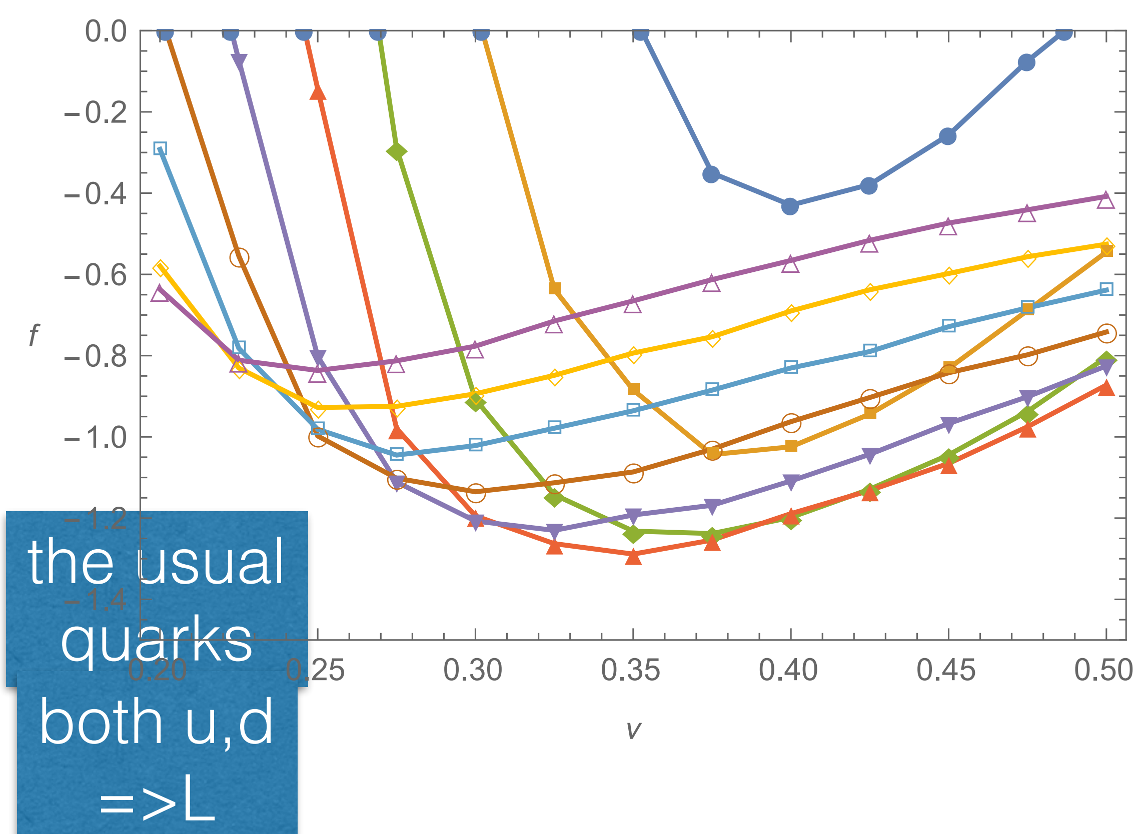
In this case each dyon type has one zero mode with one quark flavor =>N independent topological ZMZ’s!

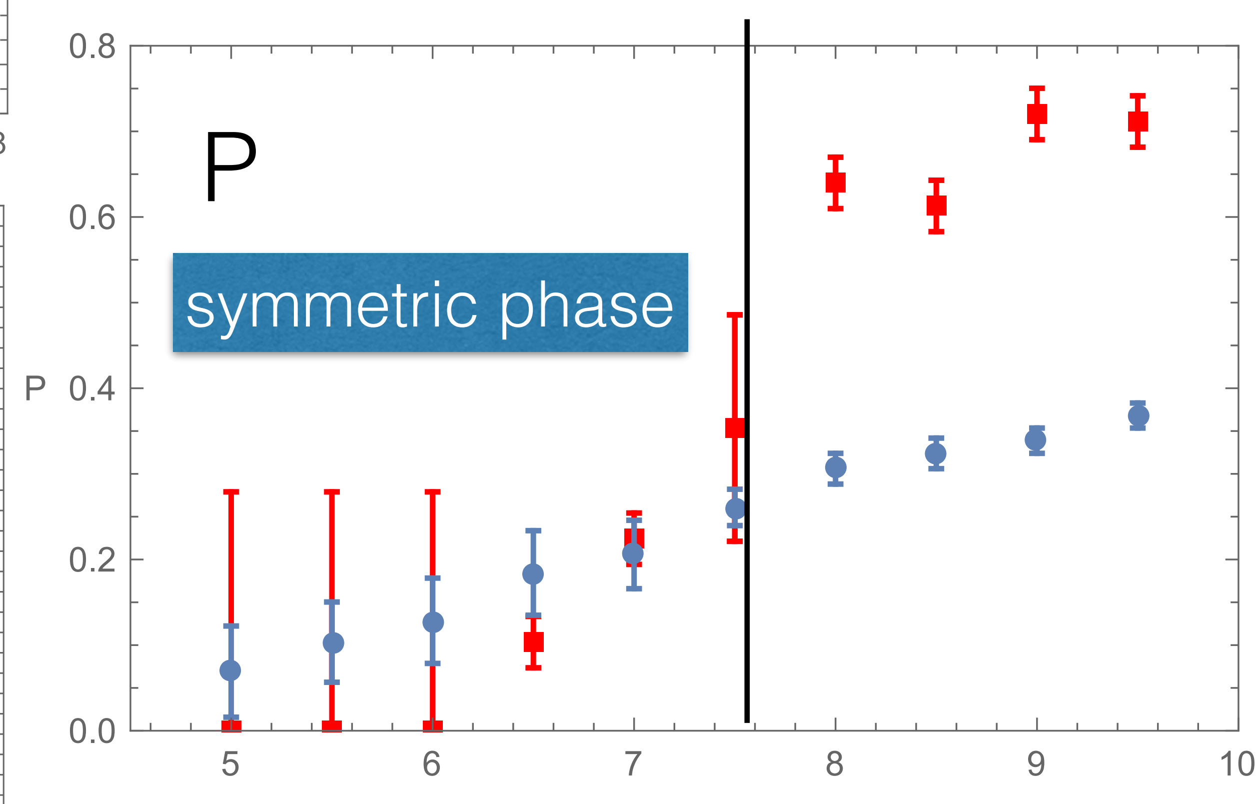
Instanton-dyon Ensembles III: Exotic Quark Flavors

Rasmus Larsen and Edward Shuryak



confining phase
gets much more
robust:
transition strong first order
mixed phase (flat F)
is observed at medium densities





$$S = (\frac{11N_c}{3} - \frac{2N_f}{3})log(\frac{T}{\Lambda_T}).$$

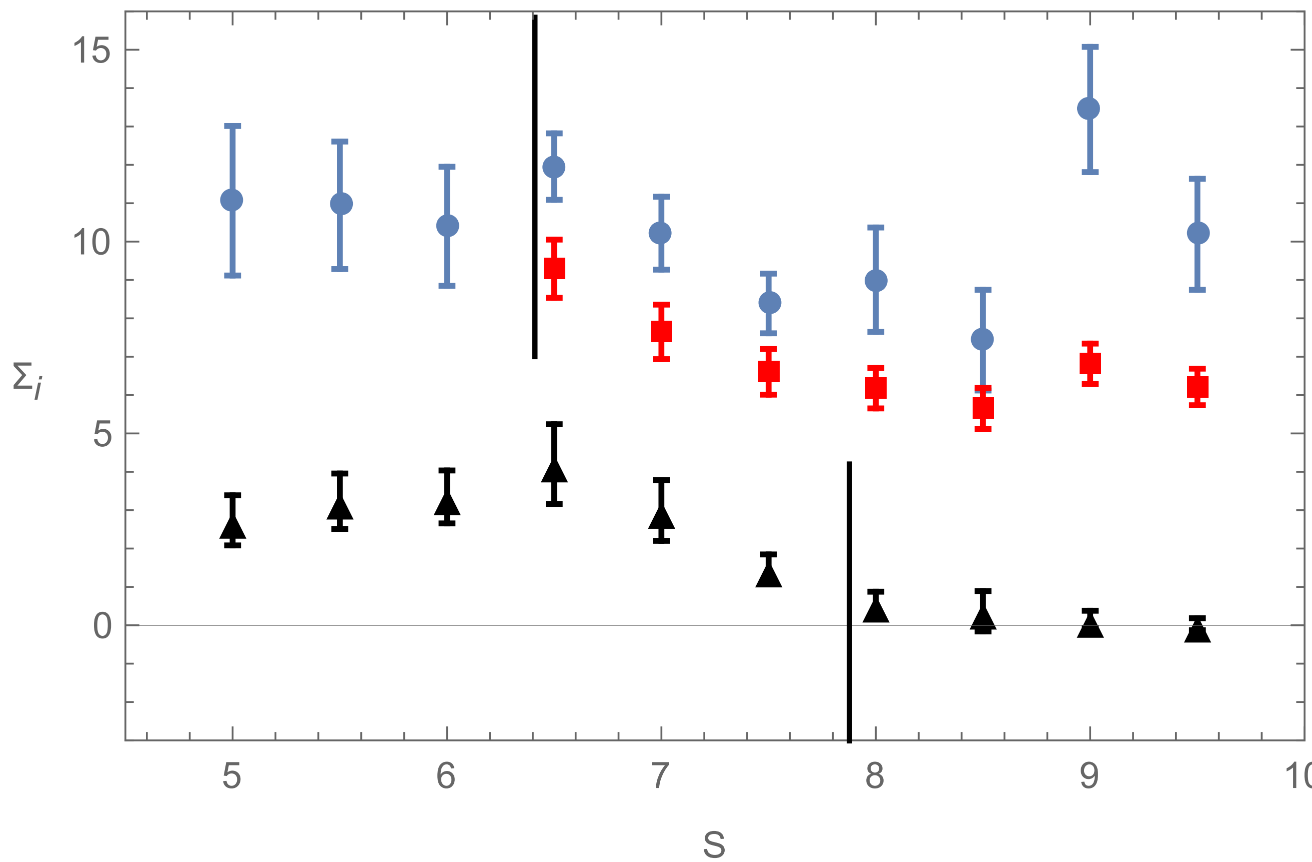
chiral symmetry breaking is dramatically different

symmetric phase

u=>M

d=>L

$$< \bar{u}u > \neq < \bar{d}d >$$



Z2 QCD

has symmetric and asymmetric phases yet apparently no chiral symmetry restoration at any T

the usual QCD has chiral restoration

FIG. 6: Chiral condensate generated by u quarks and L dyons (red squares) and d quarks interacting with M dyons (blue circles) as a function of action S, for the Z_2-symmetric model. For comparison we also show the results from II for the usual QCD-like model with $N_c = N_f = 2$ by black triangles.

why can the quark condensate be much larger for Z2?

the first lattice study of Z3 QCD

Lattice study on QCD-like theory with exact center symmetry

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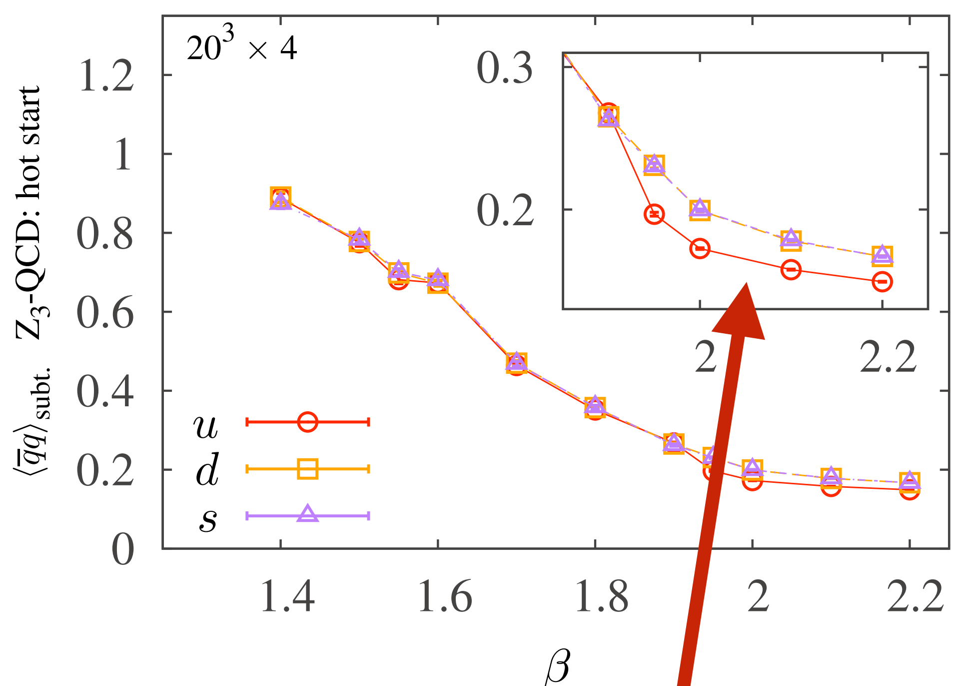


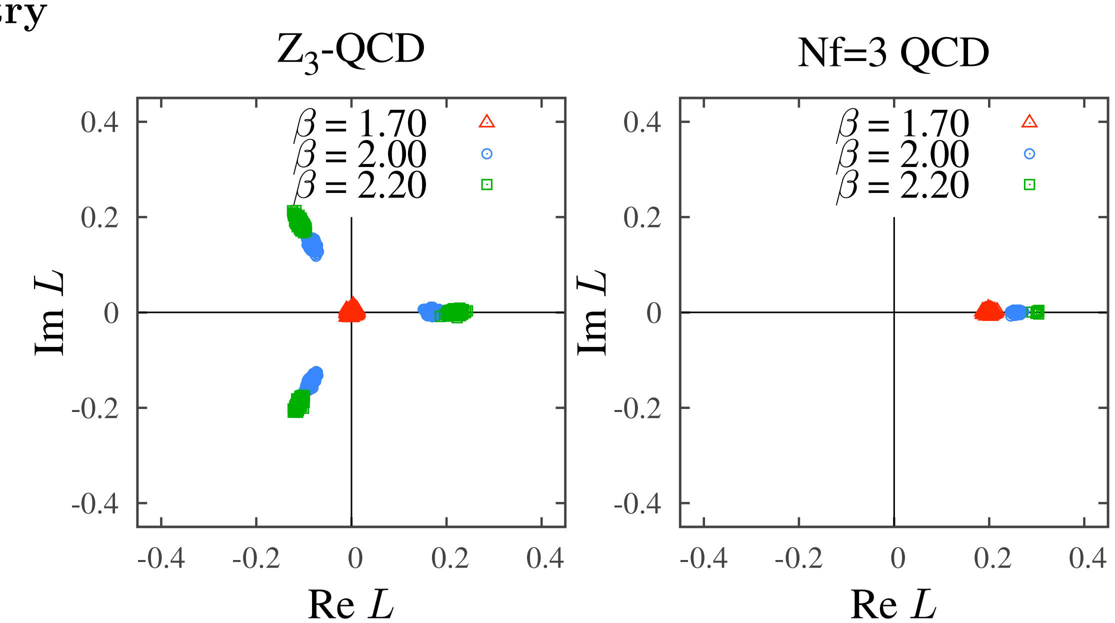
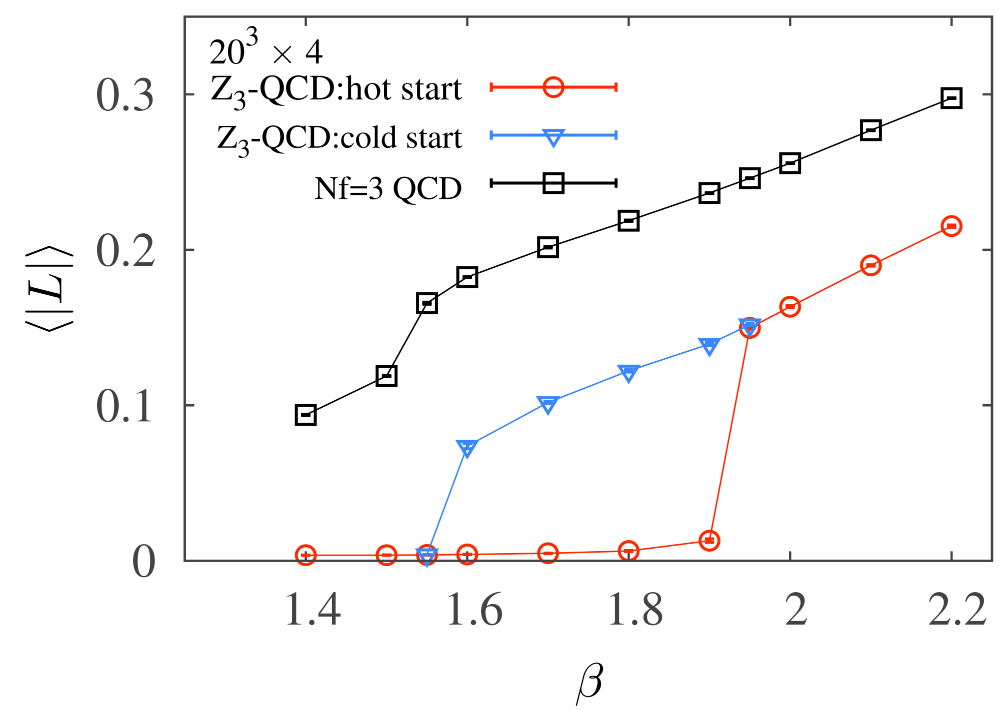


FIG. 1: Polyakov loop distribution plot in Z_3-QCD (left) and the standard three-flavor QCD (right). Based on $16^3 \times 4$ lattice for $\beta = 1.70, 2.00, 2.20$ with the same values of κ in both panels.



explanation: three flavors of quarks interact with three different ``liquids" of M1,M2,L instanton-dyons!

Summary

Instanton-dyon ensembles:
in QCD-like theories the deconfinement
and chiral transitions
are driven just by sufficiently large dyon density
=> quasicritical Tdec and Tchir are about the same

But this changes in theories with
unusual fermions.
Nontrivial flavor holonomies
(phases in boundary conditions)
dramatically change both deconfinement
and chiral transitions:
interesting dependences seen.
It is an excellent tool to fix the microscopic mechanism

Yet direct identification
of the instanton-dyons
on the lattice,
study of their density etc are
still badly needed

arXiv:1408.6563v1 [hep-ph] 27 Aug 2014

Classical interactions of the instanton-dyons with antidyons

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Instanton-dyons, also known as instanton-monopoles or instanton-quarks, are topological constituents of the instantons at nonzero temperature and holonomy. While the interaction between instanton-dyons have been calculated to one-loop order by a number of authors, that for dyon-antidyon pairs remains unknown even at the classical level. In this work we are filling this gap, by performing gradient flow calculations on a 3d lattice. We start with two separated and unmodified objects, following through the so called "streamline" set of configurations, till their collapse to perturbative fields.

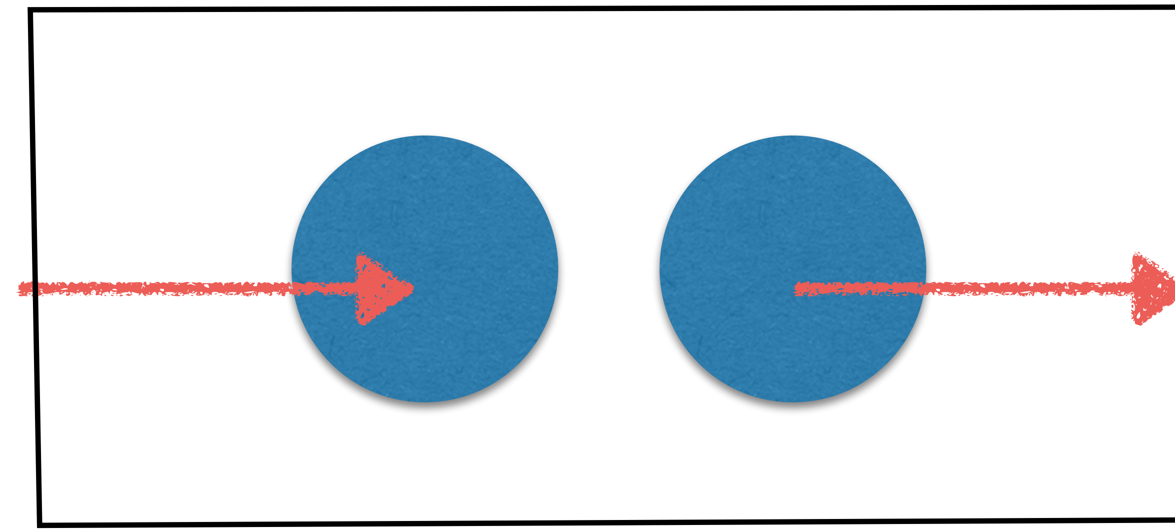
M Mbar pair on a 3d lattice (not periodic)
start with a "combed" sum ansatz and then do action gradient flow
=>"streamline configurations" found,
total magnetic charge =0
=> only Dirac string is left
total electric charge =2 (unlike instanton-antiinstanton pair)
=> massive charged gluons leave the box

Let us remind that the gauge action can be expressed in terms of the 3-dimensional action

$$S = \frac{1}{g^2}\int_0^{1/T} dx_4 S_3 = \frac{S_3}{g^2 T} \tag{25}$$

which itself scales as $S_3 \sim v$: thus the M dyon action is $\sim v/T$. We do not care about T and the gauge coupling g since it is just an overall factor in the action, and work with the S_3 itself. Furthermore, since our classical 3d theory is invariant under the transformation $A_\mu \to vA_\mu$ and $r \to vr$, the absolute units are unimportant and we can work with $v = 1$.

The gradient flow process was found to proceed via the following stages:
(i) *near initiation*: starting from relatively arbitrary ansatz one finds rapid disappearance of artifacts and convergence toward the streamline set
(ii) following the *streamline itself*. The action decrease at this stage is small and steady. The dyons basically approach each other, with relatively small deformations: thus the concept of an interaction potential between them makes sense at this stage
(iii) a *metastable state* at the streamline's end: the action remains constant, evolution is very slow and consists of internal deformation of the dyons rather than further approach
(iv) *rapid collapse* into the perturbative fields plus some (pure gauge) remnants



Dirac strings setting

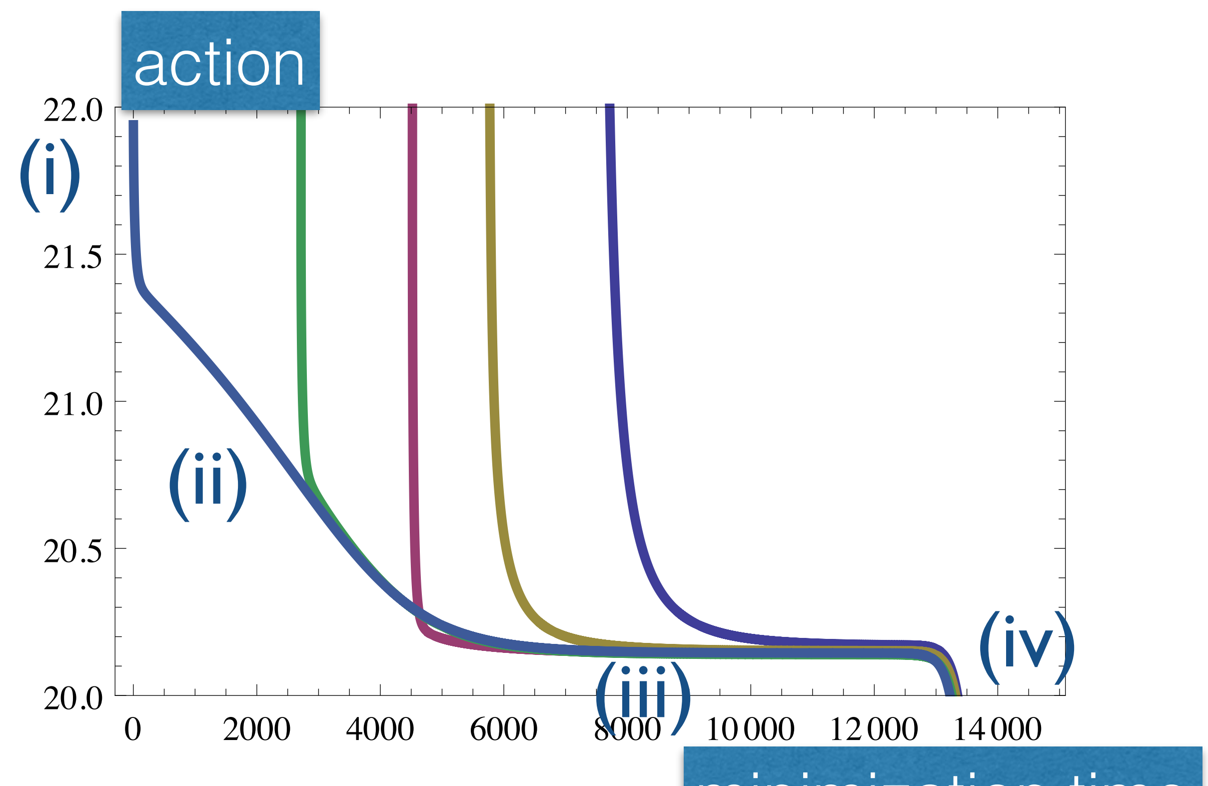


FIG. 2: Action for $v = 1$ as a functio[...] units of iterations of all links) for a separation $|r_M - r_{\bar{M}}|v = 0$, 2.5, 5, 7.5, 10 between the M and $\bar{M}$ dyon from right to left in the graph. The action of two well separated dyons is 23.88.

a stream, a pool, and then waterfall observed

Confining Dyon-Anti-Dyon Coulomb Liquid Model I

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(Dated: April 9, 2015)

We revisit the dyon-anti-dyon liquid model for the Yang-Mills confining vacuum discussed by Diakonov and Petrov, by retaining the effects of the classical interactions mediated by the streamline between the dyons and anti-dyons. In the SU(2) case the model describes a 4-component strongly interacting Coulomb liquid in the center symmetric phase. We show that in the linearized screening approximation the streamline interactions yield Debye-Huckel type corrections to the bulk parameters such as the pressure and densities, but do not alter significantly the large distance behavior of the correlation functions in leading order. The static scalar and charged structure factors are consistent with a plasma of a dyon-anti-dyon liquid with a Coulomb parameter $\Gamma_{D\bar{D}} \approx 1$ in the dyon-anti-dyon channel. Heavy quarks are still linearly confined and the large spatial Wilson loops still exhibit area laws in leading order. The t′ Hooft loop is shown to be 1 modulo Coulomb corrections.

$$\mathcal{Z}_{D\bar{D}}[T] \equiv \sum_{[K]} \prod_{i_L=1}^{K_L} \prod_{i_M=1}^{K_M} \prod_{i_{\bar{L}}=1}^{K_{\bar{L}}} \prod_{i_{\bar{M}}=1}^{K_{\bar{M}}} \times \int \frac{f d^3x_{Li_L}}{K_L!} \frac{f d^3x_{Mi_M}}{K_M!} \frac{f d^3y_{\bar{L}i_{\bar{L}}}}{K_{\bar{L}}!} \frac{f d^3y_{\bar{M}i_{\bar{M}}}}{K_{\bar{M}}!} \times \det(G[x]) \det(G[y])\, e^{-V_{D\bar{D}}(x-y)} \tag{10}$$

$$\ln Z_{1L}/V_3 = -\mathcal{V} - \frac{1}{2}\int \frac{d^3p}{(2\pi)^3} \ln\left|1 - \frac{V^2(p)}{16}\frac{p^8M^4}{(p^2+M^2)^4}\right| \tag{30}$$

with $V(p)$ the Fourier transform of (12)

$$V(p) = \frac{4\pi}{p^2}\int_0^\infty dr \sin r\, V_{D\bar{D}}(r/p) \tag{31}$$

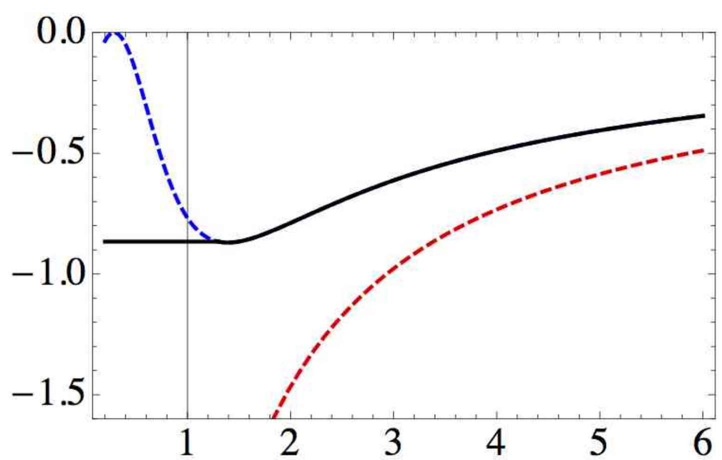
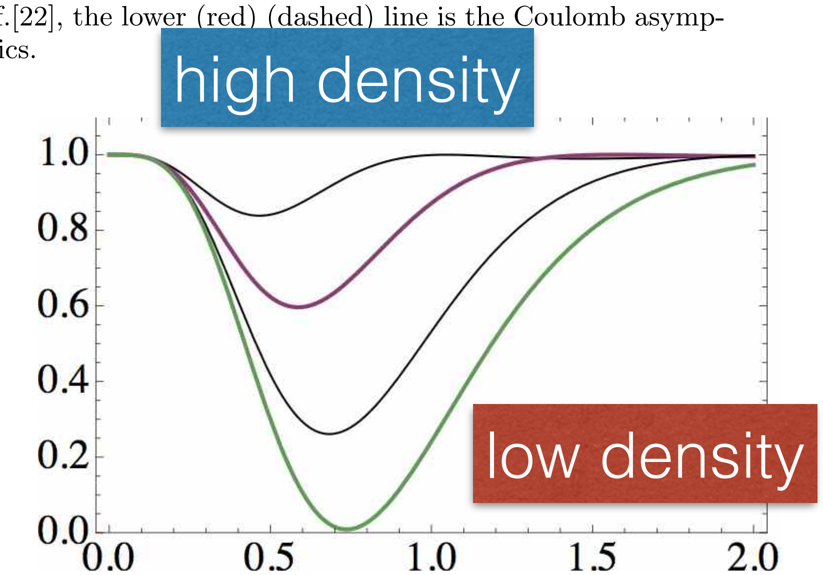


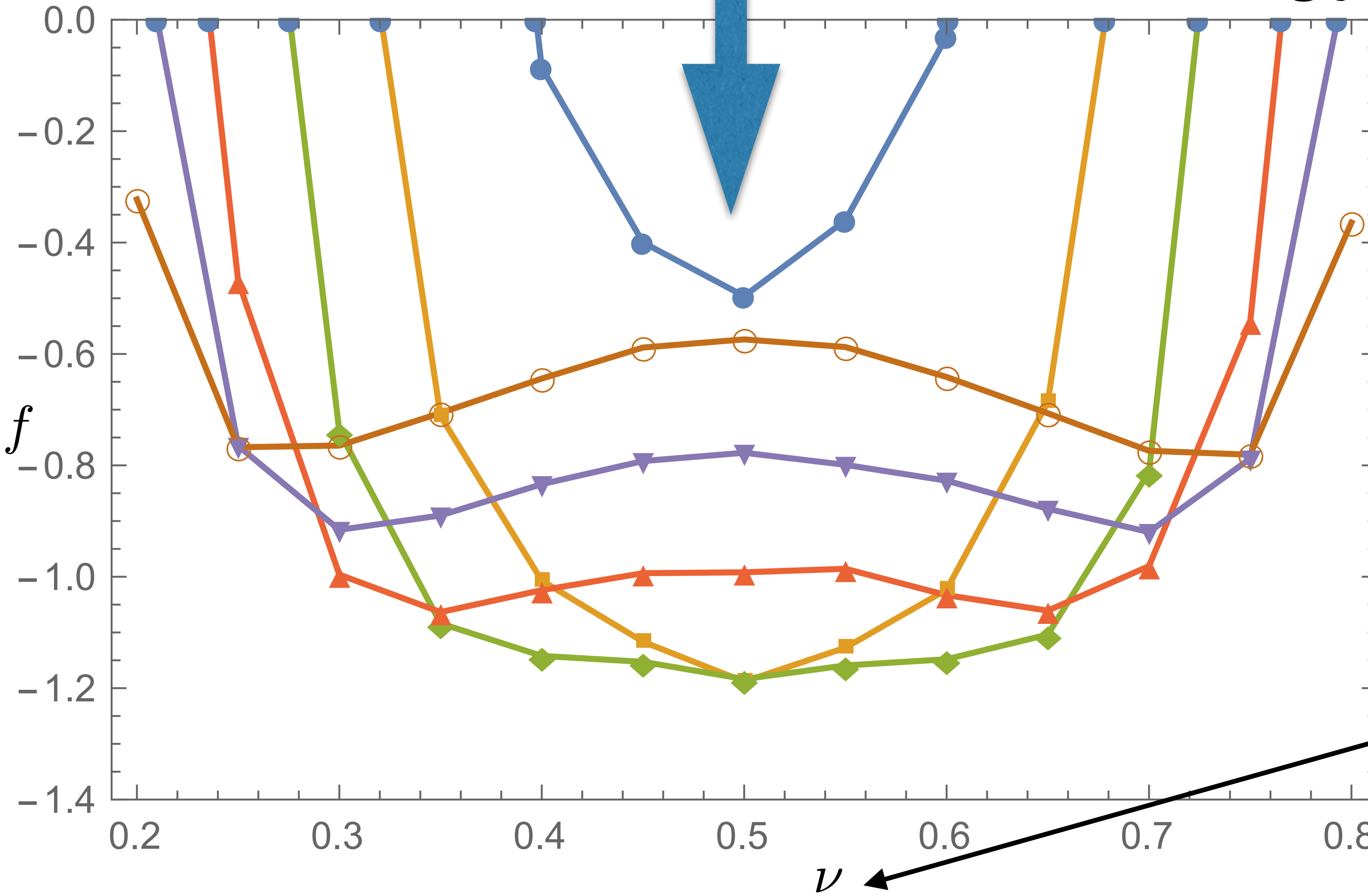
FIG. 1: (Color online) Black solid line is the SU(2) $D\bar{D}$ (dimensionless) potential versus the distance r (in units of $1/T$). Upper (blue) dashed line is the parameterization proposed in Ref.[22], the lower (red) (dashed) line is the Coulomb asymptotics.



Mean field theory can only be used at high enough dyon density or T<Tc

confined

free energy vs holonomy



$$\langle A_4^3 \rangle = v\frac{\tau^3}{2} = 2\pi T\nu\frac{\tau^3}{2}$$

$$< P >= cos(\pi\nu) \to 0$$

$$if\ \nu = 1/2$$

holonomy

$$\nu = 0\ is\ the\ trivial\ case$$

$$\nu = 1/2\ confining$$

So, as a function of the dyon density the potential changes its shape and confinement takes place

Light Quarks in the Screened Dyon-Anti-Dyon Coulomb Liquid Model II

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We discuss an extension of the dyon-anti-dyon liquid model that includes light quarks in the dense center symmetric Coulomb phase. In this work, like in our previous one, we use the simplest color SU(2) group. We start with a single fermion flavor $N_f = 1$ and explicitly map the theory onto a 3-dimensional quantum effective theory with a fermion that is only $\mathrm{U}_V(1)$ symmetric. We use it to show that the dense center symmetric plasma develops, in the mean field approximation, a nonzero chiral condensate, although the ensuing Goldstone mode is massive due to the $\mathrm{U}_A(1)$ axial-anomaly. We estimate the chiral condensate and σ, η meson masses for $N_f = 1$. We then extend our analysis to several flavors $N_f > 1$ and colors $N_c > 2$ and show that center symmetry and spontaneous chiral symmetry breaking disappear simultaneously when $x = N_f/N_c \geq 2$ in the dense plasma phase. A reorganization of the dense plasma phase into a gas of dyon-antidyon molecules restores chiral symmetry, but may preserve center symmetry in the linearized approximation. We estimate the corresponding critical temperature.

The main issue discussed in this paper is the behavior (pairing or collectivization) of the fermionic zero modes into what is called in the literature the "Zero Mode Zone" (ZMZ). The approximations used in its description follows closely the construction, developed for instantons and described in detail in refs [14]. The fermionic determinant can be viewed as a sum of closed fermionic loops connecting all dyons and antidyons. Each link – or hopping – between L-dyons and $\bar{\mathrm{L}}$-anti-dyons is described by the elements of the "hopping chiral matrix" $\tilde{\mathbf{T}}$

$$\tilde{\mathbf{T}}(x,y) \equiv \begin{pmatrix} 0 & \mathbf{T}_{ij} \\ -\mathbf{T}_{ji} & 0 \end{pmatrix} \tag{9}$$

with dimensionality $(K_L + K_{\bar{L}})^2$. Each of the entries in $\mathbf{T}_{ij}$ is a "hopping amplitude" for a fermion between an L-dyon and an $\bar{\mathrm{L}}$-anti-dyon, defined via the zero mode φ_D of the dyon and the zero mode $\varphi_{\bar{D}}$ (of opposite chirality) of the anti-dyon

$$\mathbf{T}_{ij} \equiv \mathbf{T}(x_i - y_j) = \int d^4z\, \varphi^\dagger_{\bar{D}}(z - x_i) i(\gamma\cdot\partial)\varphi_D(z - y_j) \tag{10}$$

$$\int \frac{d^3p}{(2\pi)^3} \frac{M^2(p)}{p^2 + M^2(p)} = \frac{n_D}{4}$$

gap eqn for lambda

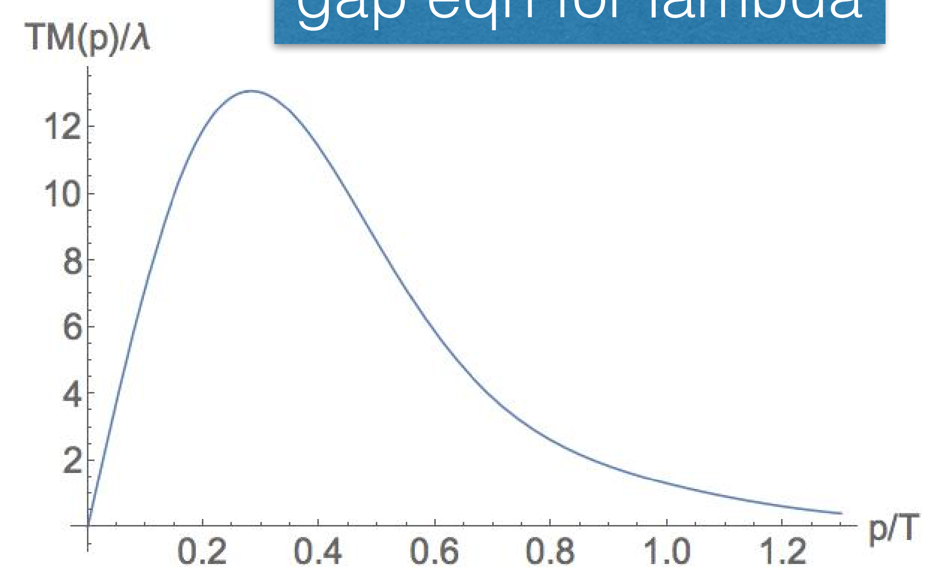


FIG. 1: The momentum dependent quark constituent mass $TM(p)/\lambda$ versus p/T.

chiral symmetry breaking for different Nf

Nc=2,Nf=1 solution is studied in detail

$$\frac{|\langle \bar{q}q \rangle|}{T^3} \approx 1.25 \left(\frac{n_D}{T^3}\right)^{1.63}$$

For general $x = N_f/N_c$, the saddle point equation in Σ of (85) gives

$$\Sigma = \left(\frac{\tilde{\lambda}}{2x\alpha(N_c)}\right)^{\frac{1}{x-1}} \quad (88)$$

after the shift $-i\lambda \to \lambda$ and $\tilde{\lambda} = N_f\lambda$. With this in mind and inserting (88) into (85) yields

$$-\mathcal{V}/\mathbb{V}_3 = -2\alpha(N_c)(x-1)\left(\frac{\tilde{\lambda}}{2x\alpha(N_c)}\right)^{\frac{x}{x-1}} + xN_c\int\frac{d^3p}{(2\pi)^3}\ln\left(1+\frac{\tilde{\lambda}^2}{N_f^2}\mathbf{T}^2(p)\right) \quad (89)$$

The effective potential (89) has different shapes depending on the ratio of the number of flavors to the number of colors x. Let us explain that in details for four cases:

(i) If $x < 1$ the first term in (89) has a positive coefficient and a negative power, so it is decreasing at small $\tilde{\lambda}$. At large value of $\tilde{\lambda}$ the second term is growing as $\ln\tilde{\lambda}$. Thus a minimum in between must exist. This minimum is the physical solution we are after.

(ii) If $1 < x < 2$ the coefficient of the first term is negative but its power is now positive. So again there is a decrease at small $\tilde{\lambda}$ and thus a minimum.

(iii) If $x > 2$ the leading behavior at small $\tilde{\lambda}$ is now dominated by the second term which goes as $\tilde{\lambda}^2$ with positive coefficient. One may check that the potential is monotonously increasing for any $\tilde{\lambda}$ with no extremum. There is no gap equation, which means chiral symmetry cannot be broken in the mean-field approximation.

(iv) If $x = 2$ there are two different contributions of opposite sign to order $\tilde{\lambda}^2$ at small $\tilde{\lambda}$. An extremum forms only if the following condition is met

$$\int\frac{d^3p}{(2\pi)^3}\mathbf{T}^2(p) < \frac{N_c}{4\alpha(N_c)} = \mathcal{O}\left(\frac{1}{N_c}\right) \quad (90)$$

Using the exact form (13) and the solution to the gap equation at $T = T_0$, we have

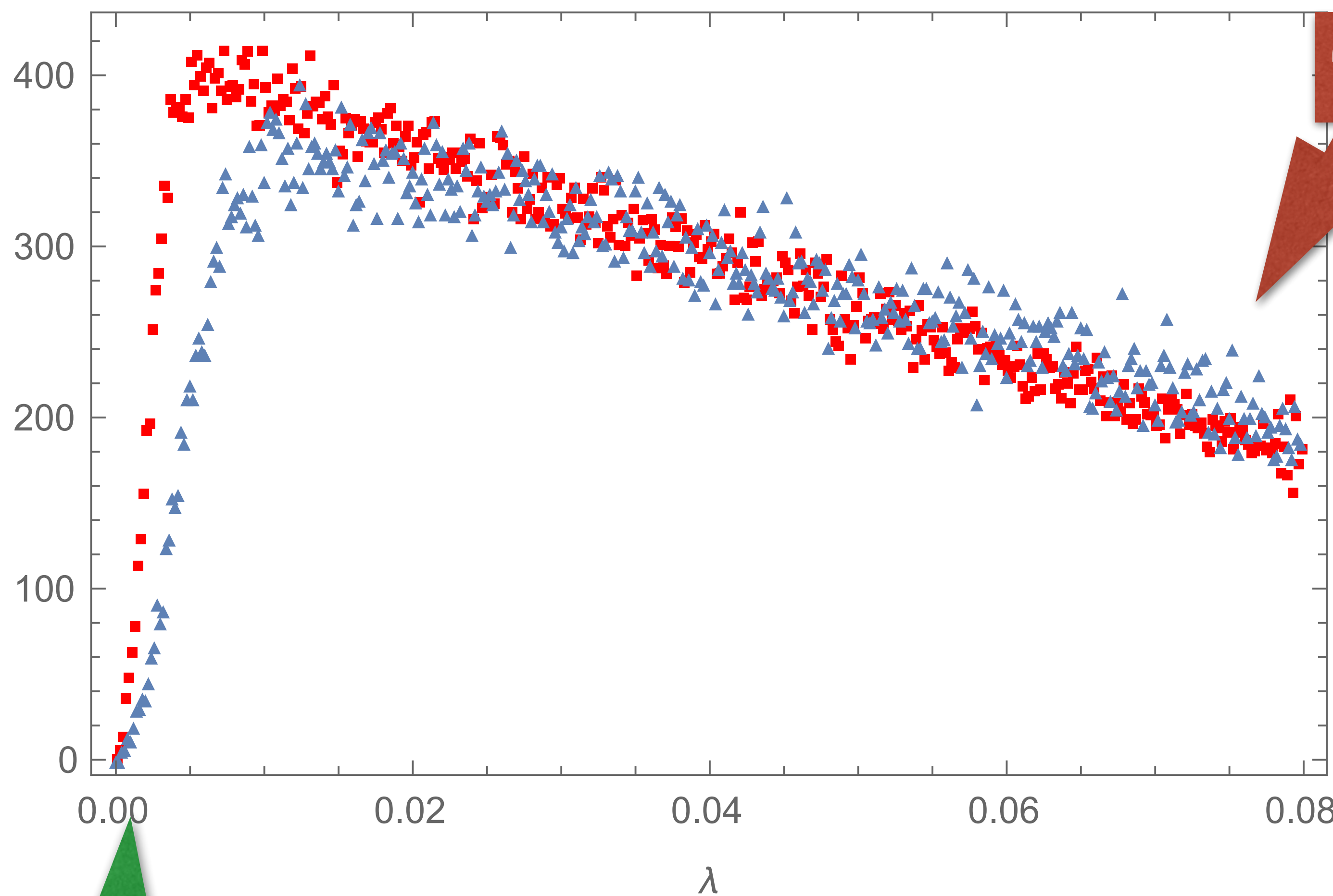
$$\int\frac{d^3p}{(2\pi)^3}\mathbf{T}^2(p) = \frac{10.37}{T_0} \quad (91)$$

which shows that (90) is in general upset, and this case does *not* possess a minimum.

Critical Nf/Nc=2 for mean field treatment

lattice: Nc=3,Nf=4 broken, NF=8 probably not

64 and 128 dyons



"inverse cusp"
is the unmistakable
sign of Nf=1 theory

$$\rho(\lambda) \sim |\lambda|(N_f^2 - 4)$$

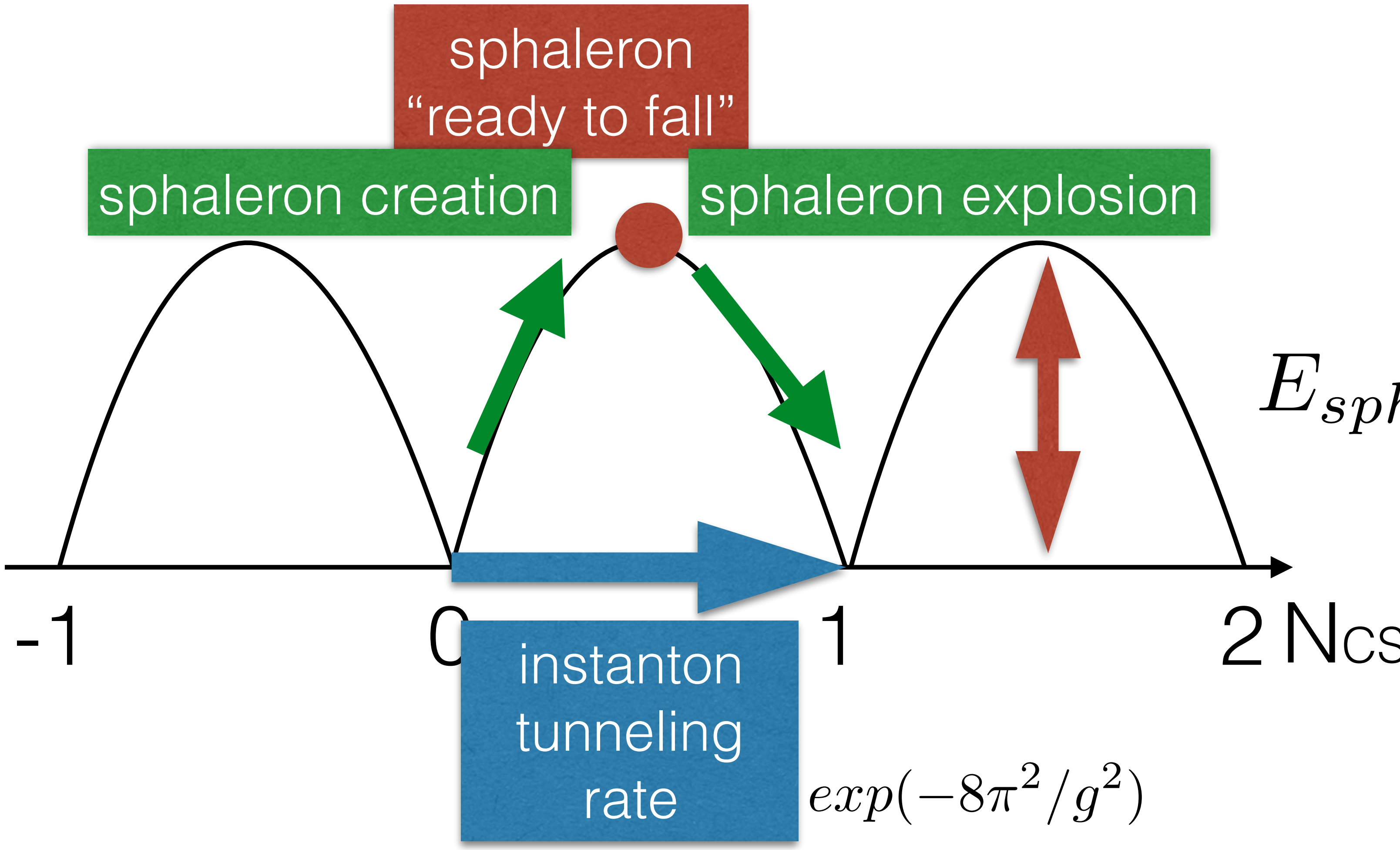
Smilga,Stern
Verbaarschot

"finite size dip"
scales as 1/V

QCD =>
one copy of the (Nf=Nc) ensemble

ZN-symmetric model =>
N copies of the (Nf=1) ensembles

anomaly, topology, instantons, sphalerons and their explosion



$$E_{sph} = \frac{3\pi^2}{g^2\rho}$$

COS sphaleron size

$$R \sim exp(-E_{sph}/T)$$

thermal excitation rate

$$\frac{1}{32\pi^2} G_{\mu\nu}\tilde{G}_{\mu\nu} = \partial_\mu K_\mu$$

anomaly =>

$$\sim \partial_\mu j^B_\mu$$

$$K_\mu = \frac{\epsilon^{\mu\alpha\beta\gamma}}{16\pi^2}(A^a_\alpha \partial_\beta A^a_\gamma + \frac{1}{3} f^{abc} A^a_\alpha A^b_\beta A^c_\gamma)$$

$$B = \int d^3x j^B_0$$

$$N_{CS} = \int d^3x K_0$$

Chern-Simons and baryon number are locked!

each transition creates 9 quarks and 3 leptons, B=L=3